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An Improved Design of Waveguide Band-Rejection Filters

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The usual design technique for waveguide band-rejection filters uses narrow-band approximations and thus discrepancies generally exist between the designed and measured response, particularly in the fairly wide passband. Nevertheless, this design technique has been used because of its simplicity and because the filter configurations obtained are relatively simple. Lately, a new design technique using transmission line synthesis became available which, theoretically, would yield the desired response. However, the physical realization results in a complicated configuration which leads to certain practical problems. This paper presents a modified technique which simplifies the structure without sacrificing performance. With this modification the design procedure becomes very simple and many of the practical problems can be avoided. This paper gives precise design information and convenient design formulas. Furthermore, it shows that excellent agreement between the designed and measured response can be achieved.

I. INTRODUCTION

The microwave waveguide band-rejection filter (BRF) now used in many radio systems has many undesirable features. The designer finds that the actual bandwidth is consistently narrower than the designed value, and tbat the filter has a unique passband VSWR behavior which becomes worse as the frequency goes farther away from the

stopband. This becomes quite a severe problem in filters designed for the high frequency or low frequency channels of the band. (For example, in the 4 GHz band,* the return loss of a filter with the stopband at 3710 MHz becomes progressively poorer as the frequency approaches 4190 MHz, and vice versa). In addition, the midband frequency of the VSWR curve and of the corresponding delay curve is found to differ from the midband frequency of the insertion loss curve at which the filter is tuned. The offset of a typical 3-cavity maximally flat BRF at the 4 GHz band with 3 dB points at ± 17 MHz can be as much as 2 MHz. Such an uncontrolled shift causes extreme difficulties in the delay equalization of a radio system.

The present BRF design is based on the lumped-element low-pass prototype filter design technique.[1] After making a proper frequency transformation, the loaded Q of each cavity can be computed. The cavities are then separated by waveguide lengths which give an effective spacing of an odd multiple of quarter wavelengths at the midband frequency, which is a standard technique to realize ladder filters in waveguide. It is desirable to keep the spacings as small as possible, but, in order to avoid higher order mode coupling between cavities, it has been determined that a three-quarter wavelength spacing is required.

In a recent investigation, it was determined that neglecting the frequency-dependent phase shift of the connecting lines results in both the narrower bandwidth and the problem of high VSWR in the passband. A rigorous analysis was carried out for a three-cavity BRF. The result shows that when the frequency dependence of the phase shift of the connecting lines is accounted for, an exact control of bandwidth is possible and the passband VSWR is improved; but the desired characteristic of the passband VSWR cannot be obtained as long as the present form of construction is retained.

Various techniques for the exact synthesis of transmission line filters have been available for many years.[2] However, they often are found to be not too practical for the design of waveguide filters because of the large number of changes of characteristic impedances usually required. There is already a sufficiently large number of discontinuities in a waveguide filter which are necessary to form the cavities. Additional discontinuities with their associated fringing susceptances, such as would be needed to change the characteristic impedance, would aggravate the practical realization difficulties. If these problems

* 3.7 GHz to 4.2 GHz

could be overcome, the transmission line synthesis would offer the advantage of being exact.

The possibility of constructing a waveguide BRF using transmission line synthesis was mentioned by Schiffman and Matthaei.[3] Unfortunately, the form they suggest involves three impedance changes in each connecting line and therefore is not practical. This paper presents an improved form with only one impedance change in each connecting line. The susceptance discontinuity present at each impedance change is absorbed very naturally in the structure without causing any design difficulty. Design formulas for filters of up to five cavities are given in a very convenient form.

The problem of the midband frequency shift between the VSWR and the insertion loss was found to be the direct consequence of inaccurate correction for the connecting lines. The precise design procedure presented here guarantees the coincidence of the midband frequency of the VSWR and insertion loss characteristic.

An experimental model was built and tested, and the measured result agrees very well with the theoretical prediction. With this design technique, it is possible to achieve over 40 dB return loss across the passband and to control the exact bandwidth and the midband frequency. A pair of identical BRF were designed and in conjunction with two hybrids, a constant resistance channel separation network was built. Over 35 dB return loss across the band was observed; in addition, the delay distortion of the dropped channel was symmetrical with respect to the midband frequency. Channel separation networks built with BRFs designed with the old techniques always had unsymmetrical delay distortion for the dropped channel.

II. BRF CONFIGURATIONS OBTAINED BY TRANSMISSION LINE FILTER SYNTHESIS

The easiest way to synthesize a transmission line filter is to use the known low-pass prototype filters as done by Schiffman and Matthaei.[3] The reactive elements of the prototype circuit are replaced by their equivalent transmission line stubs (all of the same length l and of the same propagation constant β) using the frequency transformation:

$$\omega = A \tan \beta l \tag{1}$$

where ω is the normalized frequency of the prototype circuit and A is a constant that determines the bandwidth of the filter. For a BRF, l

can be chosen to be any odd multiple of a quarter wavelength of the midband frequency. Schiffman and Matthaei chose $l = \lambda_{g0}/4$. Then, the shunt susceptances and series reactances of the prototype filter transform as follows:

$$\omega C_i = C_i A \tan\left(\frac{\pi}{2}\frac{\lambda_{g0}}{\lambda_g}\right). \tag{2}$$

$$\omega L_i = L_i A \tan\left(\frac{\pi}{2}\frac{\lambda_{g0}}{\lambda_\epsilon}\right). \tag{3}$$

It is seen that the right side of (2) corresponds to the input susceptance of an open-circuited stub of characteristic admittance

$$Y_i = C_i A$$

and that the right side of (3) corresponds to the input reactance of a short-circuited stub of characteristic impedance

$$Z_i = L_i A.$$

As shown in Fig. 1, a three-element low-pass prototype filter becomes a transmission line with three stubs using the transformation (1). These stubs are connected to the main transmission line at the same point, which would be impossible in a physical realization. To solve this problem, connecting lines between stubs have to be inserted by making use of Kuroda's identity. This identity allows the inter-

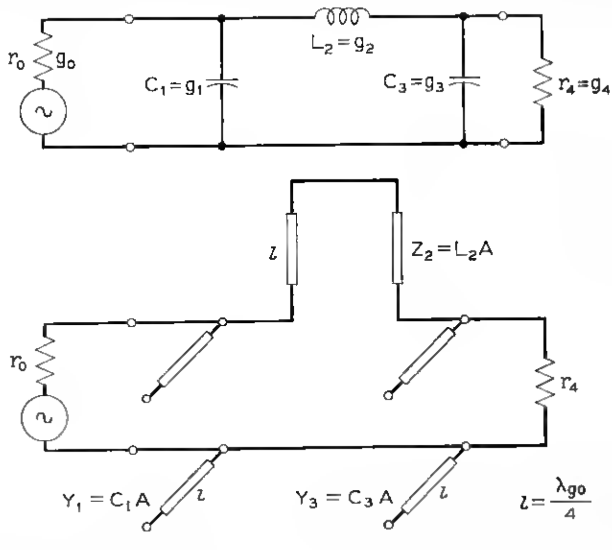

Fig. 1 — The 3 element low pass prototype filter and its corresponding BRF with transmission line stubs.

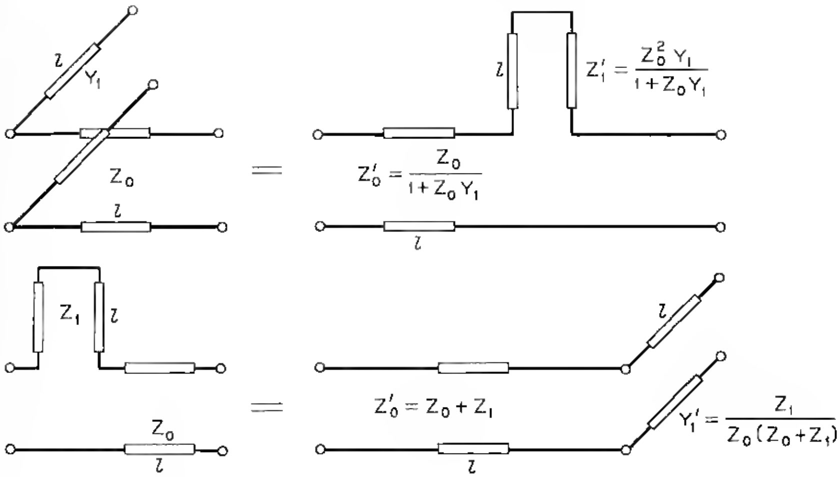

Fig. 2 — Kuroda's identity in transmission line form.

change of a stub and a connecting line (both of the same length l) as shown in Fig. 2. Adding a section of transmission line of appropriate characteristic impedance on both ends of the filter (Fig. 1) as shown in Fig. 3(a), the circuit retains its amplitude response. Applying Kuroda's identity to the two shunt open-circuited stubs, one gets the desired form in Fig. 3(b). With a proper replacement of the three series stubs by rejection cavities (only an approximation), Fig. 3(b) forms three rejection cavities in cascade with quarter wavelength lines separation between the cavities.

For the most commonly used aperture-coupled rejection cavity, a quarter wavelength separation between the cavities would not be enough because of strong coupling resulting from higher order modes between the closely spaced discontinuities. It has been found, from past experience, that a three-quarter wavelength separation is necessary in most waveguide BRFs. For this reason, Schiffman and Matthaei treated the case of three-quarter wavelength separation between cavities. Following exactly the same technique as shown in Fig. 3, one can add three pieces of quarter wavelengtb line on both sides of the stubs, as seen in Fig. 4(a), without changing the amplitude response of the filter. Then, the final form of Fig. 4b is obtained by applying Kuroda's identity three times on each side. The characteristic impedances of the connecting line sections are tabulated for two and three cavity BRF,[3] but no test result has been given in the same reference.

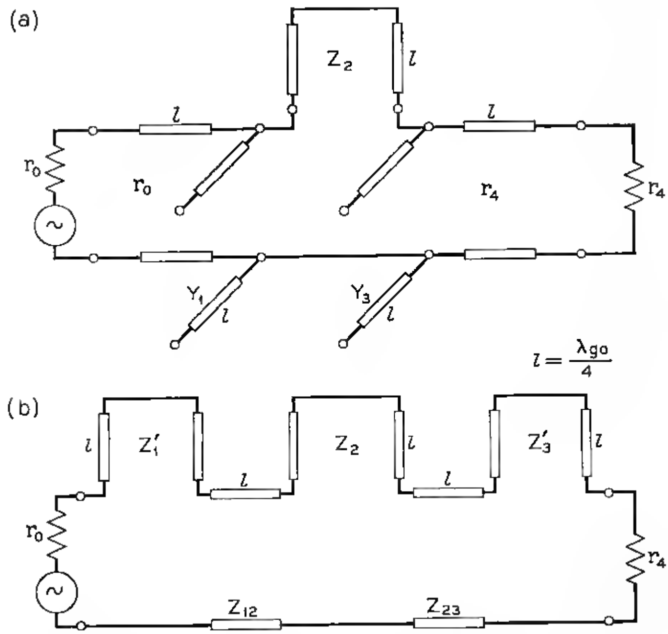

Fig. 3 — Practical realization of 3 stub transmission line BRF.

To realize the configuration of Fig. 4(b) in waveguide form, one would, usually, change the height to control the characteristic impedance such that the propagation constant β remains unchanged. A typical three cavity BRF would therefore assume the form shown in Fig. 5. Theoretically, one may be able to compensate for the discontinuity susceptances at each step by proper adjustment of the line length of each section. This, however, complicates the design procedure unduly, and it is not sure how good the result may be. As mentioned before, the higher order mode interaction between closely spaced discontinuities may degrade the performance of the filter. Since a better configuration, in every respect, is being proposed, no attention was paid to the practical problems associated with tbis suggested structure.

III. PROPOSED NEW BRF CONFIGURATION

The only restriction which must be fulfilled in presently-available transmission line synthesis is that all the line sections must have the same length l and the same propagation constant β so that the frequency transformation (1) can be applied. To construct a BRF, one

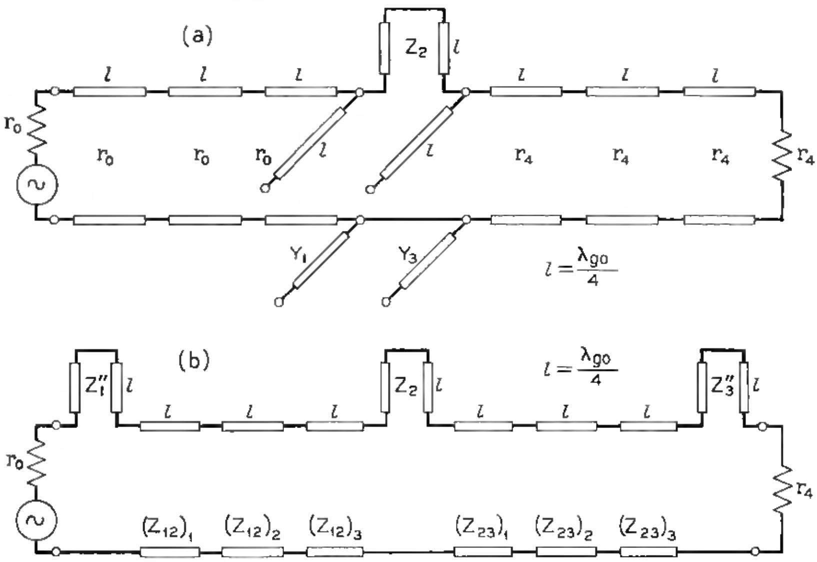

Fig. 4 — Suggested 3 stub transmission line BRF to be realized in waveguide form.

may choose the length l to be any odd number of quarter wavelength of the midband frequency. For the waveguide BRF, it is convenient to choose $l = 3\lambda_{g0}/4$. Then the shunt susceptance and series reactance of the filter become

$$\omega C_i = C_i A \tan\left(\frac{3\pi}{2}\frac{\lambda_{g0}}{\lambda_g}\right) \tag{4}$$

$$\omega L_i = L_i A \tan\left(\frac{3\pi}{2}\frac{\lambda_{g0}}{\lambda_g}\right) \tag{5}$$

and the equivalence between prototype circuit and the transmission line stubs shown in Fig. 1 still holds except that the constant A is

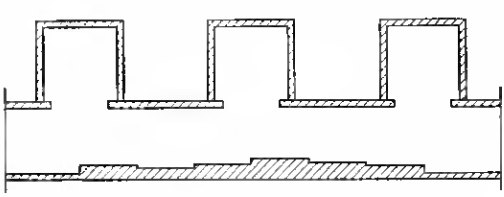

Fig. 5 — Complicated form of a 3 cavity waveguide BRF.

changed due to the change of stub length l. The value of A may be computed from the given band edge attenuation specification; for example, from the 3 dB point for maximally flat filters or from the cutoff frequency for equal ripple filters. Let ω_1 be the edge frequency of the prototype filter and λ_{g1} be the corresponding waveguide wavelength, then

$$A = \omega_1 \cot\left(\frac{3\pi}{2}\frac{\lambda_{g0}}{\lambda_{g1}}\right). \tag{6}$$

After obtaining the characteristic impedances of all the stubs, one can apply Kuroda's identity to get the same form as Fig. 3(b), of course with $l = 3\lambda_{g0}/4$. This is almost the desired form for waveguide BRF except that the series stubs have to be replaced by cavities with the proper Q values. There are many ways to find the equivalence between stubs and cavities, all of which are approximations. Among them, one method is found to be particularly convenient. The input impedance of a short-circuited stub of $3\lambda_{g0}/4$ length with characteristic impedance Z_0 is

$$Z_{in} = jZ_0 \tan\left(\frac{3\pi}{2}\frac{\lambda_{g0}}{\lambda_g}\right) \tag{7}$$

By the well known partial fraction expansion of the tangent function, one gets:

$$Z_{in} = jZ_0 \frac{4}{\pi}\left(\frac{3\lambda_{g0}}{\lambda_g}\right)\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2 - \left(\dfrac{3\lambda_{g0}}{\lambda_g}\right)^2} \tag{8}$$

In the vicinity of the resonance frequency ($\lambda_{g0}/\lambda_g \approx 1$), it is clear that the $k = 2$ term dominates (8). Then:

$$Z_{in} = jZ_0 \frac{4}{3\pi}\left[\frac{\lambda_g}{\lambda_{g0}} - \frac{\lambda_{g0}}{\lambda_g}\right]^{-1} + \cdots . \tag{9}$$

Taking this approximation, it is clear that the equivalent cavity must possess a loaded Q of

$$Q_{\text{loaded}} = \frac{3\pi}{2Z_0}. \tag{10}$$

This approximation is found to be sufficiently accurate for frequencies close to the resonance frequency as is the case for most waveguide filters.

Most of the advantages of this new configuration are obvious. As seen in Fig. 6, a three-to-one reduction in the number of steps is

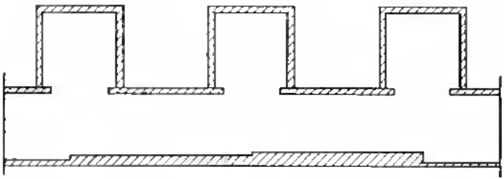

Fig. 6 — Improved form of a 3 cavity waveguide BRF.

obtained, which makes the design procedure much simpler. In addition, the spacing between discontinuities has become large, hence, one would expect less practical problems in realization (because of higher order mode interaction). One rather subtle practical advantage of this structure is that all the steps are located right under the center of the coupling apertures. For all inductive coupling apertures (the circular hole is one of them), a capacitive compensating stud is always required at the location of the hole, which in this structure happens to coincide with the location of the steps. Because of the capacitive step susceptance, the size of the compensating studs just has to be made slightly smaller. No additional compensation is needed for all the step susceptances.

IV. DESION INFORMATION

A convenient table for filters with two to five cavities is presented in this section which gives the Q values for each cavity and the characteristic impedance for each connecting line. The following symbols are used:

n = number of cavities
R_g = normalized generator impedance
R_l = normalized load impedance
Q_i = loaded Q value of ith cavity
g_i = normalized values of the low pass prototype filter
$Z_{i,i+1}$ = normalized characteristic impedance of the connecting line between the ith and the $(i + 1)$th cavity
A = bandwidth constant defined in (6)

$n = 2$

$$Q_1 = \frac{3\pi}{2R_g}\left(1 + \frac{1}{Ag_0g_1}\right) \qquad Z_{12} = R_g(1 + Ag_0g_1)^{-1}$$

$$Q_2 = \frac{3\pi}{2R_g}\frac{g_0}{Ag_2} \qquad R_l = R_g\frac{1}{g_0g_3}$$

$n = 3$

$$Q_1\ ,\ Q_2\ ,\ Z_{12}\ :\ \text{same as}\ n = 2$$

$$Q_3 = \frac{3\pi g_0}{2R_g g_4}\left(1 + \frac{1}{Ag_3g_4}\right) \qquad Z_{23} = R_g \frac{g_4}{g_0}(1 + Ag_3g_4)^{-1}$$

$$R_l = R_g \frac{g_4}{g_0}$$

$n = 4$

$$Q_1 = \frac{3\pi}{2R_g}\left(2 + \frac{1}{Ag_0g_1}\right) \qquad Z_{12} = R_g\left(\frac{1 + Ag_0g_1}{1 + 2Ag_0g_1}\right)$$

$$Q_2 = \frac{3\pi}{2R_g}\left(\frac{1}{1 + Ag_0g_1} + \frac{g_0}{Ag_2(1 + Ag_0g_1)^2}\right)$$

$$Z_{23} = R_g g_0\left(Ag_2 + \frac{g_0}{1 + Ag_0g_1}\right)^{-1}$$

$$Q_3 = \frac{3\pi}{2R_g}\frac{1}{Ag_0g_3} \qquad Z_{34} = R_g g_0 g_5 (1 + Ag_4g_5)^{-1}$$

$$Q_4 = \frac{3\pi}{2R_g}\frac{1}{g_0g_5}\left(1 + \frac{1}{Ag_4g_5}\right) \qquad R_l = R_g g_0 g_5$$

$n = 5$

$$Q_1\ ,\ Q_2\ ,\ Q_3\ ,\ Z_{12}\ ,\ Z_{23}\ :\ \text{same as}\ n = 4$$

$$Q_4 = \frac{3\pi}{2R_g g_0}\left(\frac{1}{1 + Ag_5g_6} + \frac{g_6}{Ag_4(1 + Ag_5g_6)^2}\right),$$

$$Z_{34} = R_g g_0\left(Ag_4 + \frac{g_6}{1 + Ag_5g_6}\right)^{-1}$$

$$Q_5 = \frac{3\pi}{2R_g}\frac{g_6}{g_0}\left(2 + \frac{1}{Ag_5g_6}\right) \qquad Z_{45} = R_g \frac{g_0}{g_6}\left(\frac{1 + Ag_5g_6}{1 + 2Ag_5g_6}\right)$$

$$R_l = R_g \frac{g_0}{g_6}$$

To construct connecting lines for a given characteristic impedance, one may modify the heights b_i of waveguides according to

$$\frac{Z_1}{Z_2} = \frac{b_1}{b_2}. \tag{11}$$

Information on cavity design has been available for a long time. A set of curves was plotted of loaded Q vs aperture size for different

frequencies, of cavity length vs aperature size for different frequencies, and of stud length vs aperture size for different frequencies. Such frequencies were chosen that both the 4 GHz (3.7–4.2) and the 6 GHz (5.925–6.425) bands are covered.

The equivalent circuit of a properly compensated cavity of Fig. 7(a) is known (to the extent of the approximations used here) to be of the form shown in Fig. 7(h) referred to the reference plane T. Aside

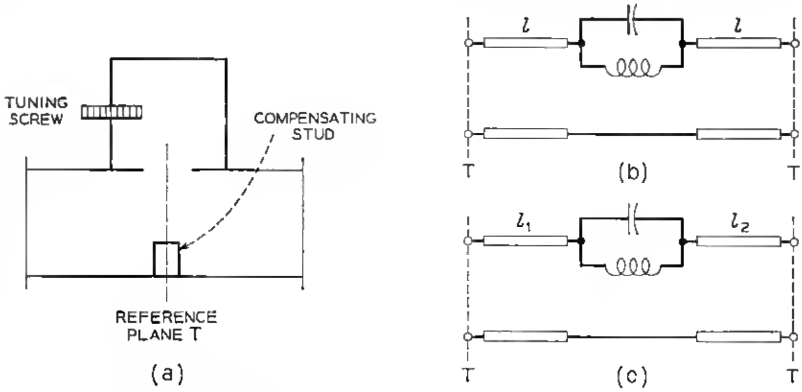

Fig. 7 — Waveguide rejection cavity and its equivalent circuits.

from the resonance circuit, an additional piece of transmission line of length l must he added at each side, in order to make the connecting line sections exactly $3\lambda_{g0}/4$ and this line element must be taken into account accurately. It was found that the unpredictable midband frequency shift between VSWR and insertion loss response, which is one of the major problems of the previously designed BRF, is the direct consequence of an inaccurate correction for this line element. A theoretical study showed that any inaccuracy in the length of tbe connecting line may result in a shift in midhand frequency of the VSWR response witb respect to the insertion loss response.* One typical 3-cavity 4 GHz BRF with $\Delta f_{3dB} = \pm 17$ MHz was found to have a 2 MHz shift with a 35 mil error in each connecting line.

Because of its importance, an attempt was made to measure accurately the values of l for various sizes of coupling aperture. The first set of data based on tbe symmetrical equivalent circuit (Fig. 7h) was very disappointing since, for a simple measurement like this, the

* Dissipation must be taken into account since this phenomenon does not exist in the lossless case.

widely spread measurement data was unexpected. It was found later that the symmetry of the structure is destroyed by the tuning screw on one side of the cavity (Fig. 7a); consequently, the symmetrical equivalent circuit 7(b) is no longer valid.

To make the equivalent circuit unsymmetrical, the circuit in Fig. 7(c), where the transmission lines on each side have different length, is introdueed. This does not solve the prohlem, however, beeause both the length l_1 and l_2 depend strongly on the amount of tuning added to the cavity. A simple plot of l_1 and l_2 does not make sense unless one could specify a fixed amount of tuning. Fortunately, there is a quantity, mainly $l_1 + l_2$, which is very insensitive to the amount of tuning. As measured in the 4 GHz band, this quantity is accurate within a variation of 2 mils for a quite large tuning range. It is believed that this would be true for the other frequency bands, too. Hence, plots of $(l_1 + l_2)/2$ vs hole diameter are presented in Figs. 8 and 9 for 4 and 6 GHz bands, respectively. For small tuning, it is true that $l_1 \approx l_2 \approx (l_1 + l_2)/2$. Furthermore, it was found that l_1, the length on the tuning screw side, is always larger than l_2, and that the difference, $l_1 - l_2$, may vary from zero to 20 mils depending on the tuning range for the sizes of apertures used in this study.

V. EXPERIMENTAL RESULTS

To verify the theoretical work discussed above and the available design data, a 3-cavity BRF was designed. Because of the approxima-

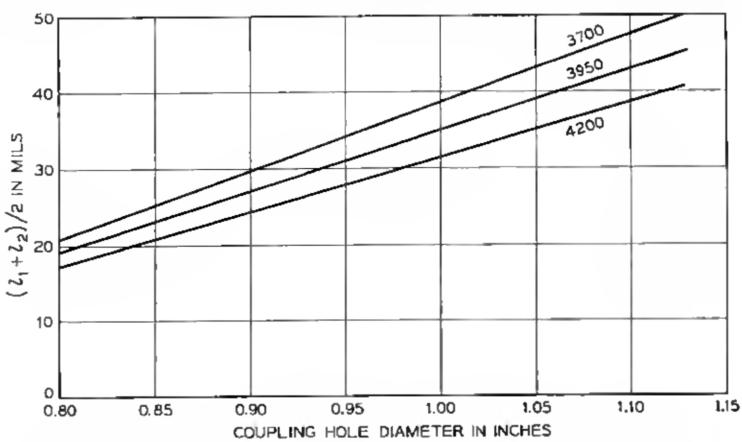

Fig. 8 — Measured information on waveguide rejection cavities in the 4 GHz band.

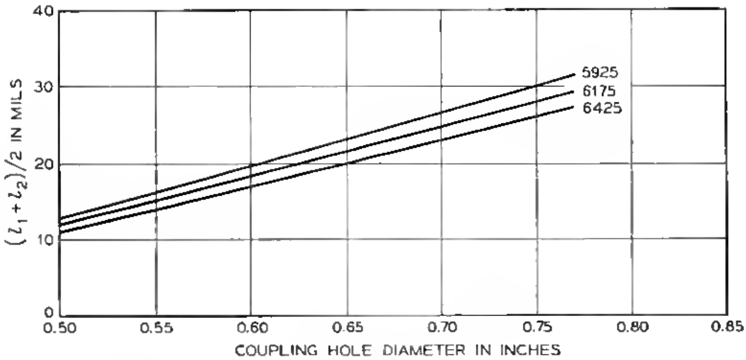

Fig. 9 — Measured information on waveguide rejection cavities in the 6 GHz band.

tions involved in the equivalence between cavity and stub line, there might still be deviations from the expected performance at frequencies well removed from the center frequency. Therefore, the filter was purposely designed to operate in the highest frequency channel (4190 MHz) of the 4 GHz band. In this case, the passband extends almost 500 MHz below the midband frequency of the filter. Any severe discrepancy caused by the approximation should show up in this extreme case. The filter was designed to have a maximally flat response with the 3 dB points at ± 17 MHz, neglecting intrinsic losses. In order to convey some idea of the "exactness" of the filter performance, the measured results are compared with the computed theoretical response. This computed curve is the theoretical response of a transmission line filter of Fig. 3(h) ($l = 3\lambda_{g0}/4$) with all line sections having an attenuation constant:

$$\alpha = 10^{-4} \text{ neper per inch.}$$

As seen in Figs. 10 and 11, the filter shows lower VSWR value in the vicinity of resonant frequency which is attributed to slightly higher intrinsic loss than the theoretical estimated value. However, over 40 dB return loss was obtained across the entire passband which is in very good agreement with the computed results.

Measured data were also taken from a channel-separating network constructed with two identical BRFs and two hybrid junctions. The channel separating network has over 35 dB return loss across the band. The delay distortion of the dropped channel was also measured, and the result shows that the delay distortion vs Δf curve is symmet-

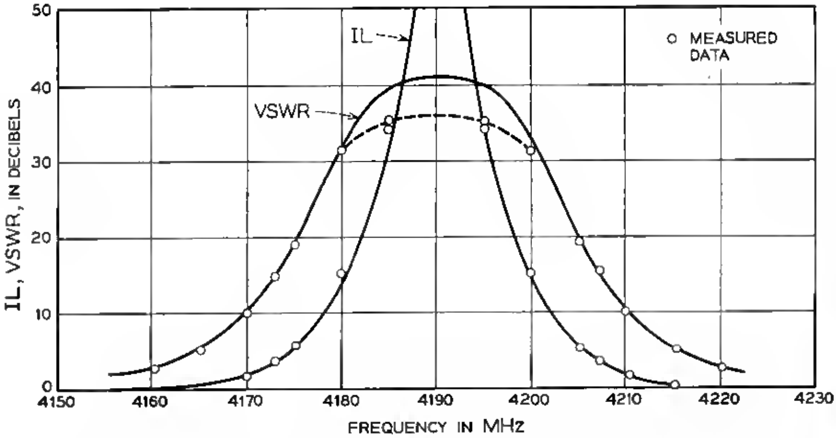

Fig. 10 — Computed and measured stop band performance of a typical 3 cavity maximally flat BRF.

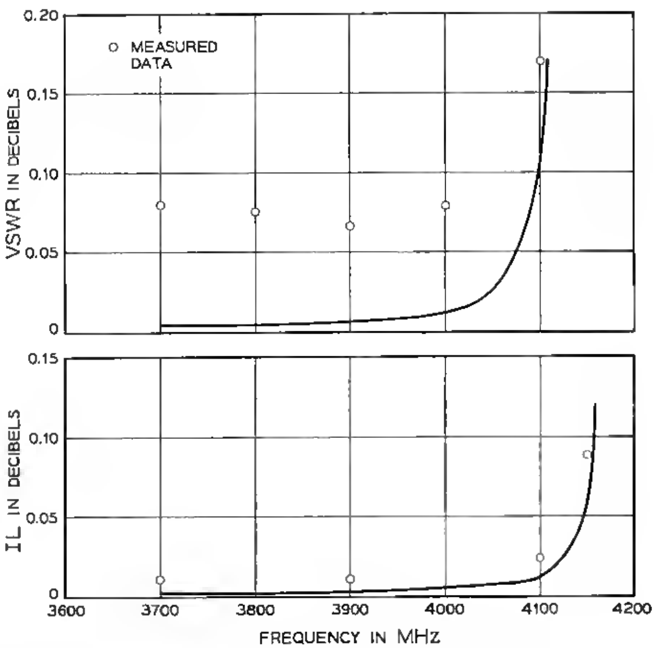

Fig. 11 — Computed and measured passband performance of a typical 3 cavity maximally flat BRF

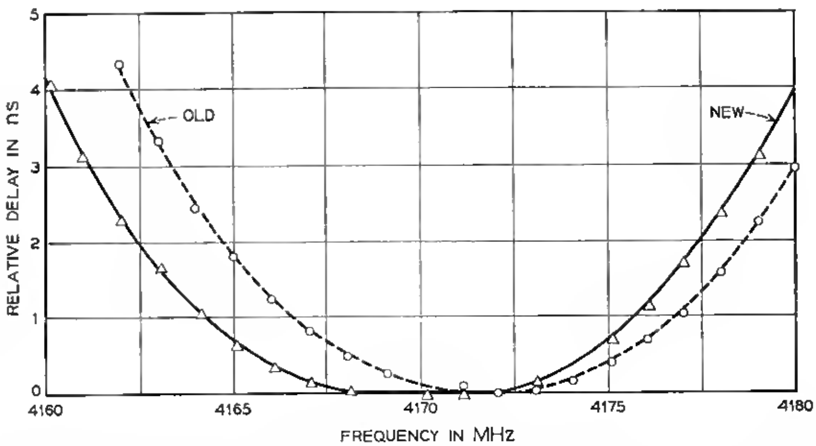

Fig. 12 — Measured delay distortion of a channel separation network using present design techniques as compared with that using previous techniques.

rical with respect to the midband frequency. The measured data is shown in Fig. 12 where the delay distortion curve of the present existing channel separating network is plotted for comparison.

VI. ACKNOWLEDGMENT

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